

## Analysis of MIMO FSO over different Modulation Techniques

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### ABSTRACT

Atmospheric turbulence is the main impairment in free space optical communication links. To mitigate the effect of turbulence spatial diversity techniques are used. In this paper, we analyse the performance of Gamma-Gamma channel model with spatial diversity and compare it with K-distribution. The modulation techniques assumed here are on-off keying, binary PPM and binary phase shift keying and the bit error rate and Gain performance with single input single output (SISO), single input multiple output (SIMO), multiple input single output (MISO) and multiple input multiple output (MIMO) are presented.

*Keywords:* Binary Phase Shift Keying (BPSK), Bit Error Rate (BER), Gamma-Gamma channel model, K-distribution model, Multiple Input Multiple Output (MIMO), Multiple Input Single Output (MISO), Single Input Multiple Output (SIMO), spatial diversity

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### INTRODUCTION

Radio frequency links have reached a saturation level with the demand for higher data rates opens up the era Free space optical communication which experimentally claims to provide higher data rates up to 160Gbps provided by Willebrand and Ghuman (2001). The visible and infrared frequency ranges used in FSO does not require license from government agencies. In FSO transmission the data is optically modulated with narrow wavelengths to provide security and privacy as explained by Bhatnagar and Ghassemlooy (2016). The main impairment in FSO is associated with atmospheric conditions and turbulence induced fading. Turbulence

may be caused by variations in temperature or turbulence induced fading creates fluctuations in phase and amplitude of a transmitted signal. Tatarskii and Zavorotnyi (1985); Khalighi and Uysal (2014) has been explained turbulence theory using Kolomogorov model by relating it with parameters such as: the refractive

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index  $\sigma_I$ , inner scale of turbulence  $l_0$  and outer scale of turbulence  $L_0$ . To describe the extent of turbulence, the scintillation index (SI) is used as standard which is defined in Eq. 1 as

$$\sigma_I^2 = E\{I^2\}/E\{I\}^2 - 1 \tag{1}$$

Where  $I$  is the irradiance and  $E\{.\}$  is expectation of irradiance,  $I$ .

The impact of turbulence of FSO channel link can be modelled using: Lognormal model, K-distribution, and Gamma-Gamma and Rayleigh distribution. While Zhu and Kahn (2002), explained that lognormal model is valid only for low turbulence condition, the K-distribution model is applicable for strong turbulence. The Gamma-Gamma is valid for all levels of turbulence as pointed out by Chatzidihamantis, Sandalidis, Karagiannidis, Kotsopoulos and Matthaïou (2010). The Gamma-Gamma model is based on doubly stochastic theory that takes into account the effect of small and large turbulence eddies. In this paper the two basic models have been considered Gamma-Gamma and K-distribution model. In section I below FSO communication is discussed. In section II system models and their mathematical notations are presented.

### System Model

FSO links are the line of sight (LOS) communication links in which optically modulated data is transmitted as physical media. The transmitters used for this purpose are lasers and light emitting diodes (LED) while to facilitate reception the data is demodulated using optical detectors. The free space optical communication provides more security than RF and it serves large users because of high bandwidth.

The received optical signal is given as equation 2:

$$y = \eta \times I + n \tag{2}$$

In equation 2  $x$  represents information bits it can be either 0 or 1. Where  $n$  is additive white Gaussian noise with mean = 0 and variance =  $N_0/2$ .  $I$  is the normalised irradiance. The optical to electrical conversion is denoted by  $\eta$ .

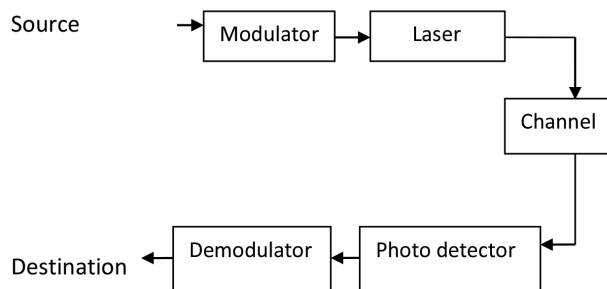


Figure 1. Basic block diagram of free space optical communication system

The losses occur in channel which are mainly misalignment losses which occurs due to building motion also known as building sway and beam wander, the reason for atmospheric losses are fog, rain etc. and atmospheric turbulence induced fading is the phenomenon which occurs due to fluctuations of signal because of variable temperature and amplitude, which degrades the performance of FSO links so it is necessary to design a channel with good performance to make the efficient communication. There are various channel models in FSO to define channel losses.

## METHODS

### Statistics of Channel Models

**Gamma-Gamma channel model.** The Gamma-Gamma channel model is widely accepted model as it is fit for all kinds of turbulence scenarios. The received irradiance ( $I$ ) estimation for this model is based on product of two gamma random processes  $I_x$  and  $I_y$  which arises from small turbulence and large turbulent eddies. The probability density function (PDF) of irradiance ( $I$ ) in Gamma-Gamma distribution is given by equation 3 given by Al-Habash, Andrews, and Phillips (2001); Yang, Gao, and Alouini (2014).

$$f(I) = \frac{2(\alpha\beta)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} I^{(\alpha+\beta)/2-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta I}), I > 0 \quad (3)$$

Where

$I$  is irradiance

$\Gamma(\cdot)$  is gamma function

$K(\alpha, \beta)$  is Bessel function of second order.

The  $\alpha$  and  $\beta$  are numbers of small and large turbulence cells and given by equation no. 4 and 5 respectively:

$$\alpha = \frac{1}{\exp\left[\frac{0.49\sigma^2}{(1+1.11\sigma^{12/5})^{7/6}}\right]-1} \quad (4)$$

$$\beta = \frac{1}{\exp\left[\frac{0.51\sigma^2}{(1+0.69\sigma^{12/5})^{5/6}}\right]-1} \quad (5)$$

Where  $\sigma^2$  represents variance.

The scintillation index (SI) which can be applied to Gamma-Gamma channel describes the impact of turbulence is given as:

$$\sigma^2 = (1/\alpha) + (1/\beta) + (1/\alpha\beta). \quad (6)$$

By considering the Gamma-Gamma channel model some special cases are derived. The K-distribution is obtained by setting  $\beta=1$  and another case in which  $\beta$  is set at infinity.

**K- distribution channel model.** The K-distribution model is valid for strong turbulence condition as explained by Tsiftsis, Sandalidis, Karagiannidis, and Uysal (2009); Kaur, Jain, and Kar (2016). The probability density function of K-distribution is obtained from equation 3 as a special case by setting  $\beta=1$ , the derived PDF is 7:

$$f(I) = \frac{2(\alpha)^{\frac{(\alpha+1)}{2}}}{\Gamma(\alpha)} I^{((\alpha-1)/2)} K_{\alpha-1}(2\sqrt{\alpha I}), I>0 \tag{7}$$

Where I is irradiance,  $\Gamma(\cdot)$  is gamma function and  $K(\alpha, 1)$  is Bessel function of second order.

**Performance Analysis of SISO Channels**

**BER Analysis.** BER (bit error rate) is the parameter used in communication to analyse the performance of those systems which transmits digital data. It is defined as the ratio of bits in error to the total transmitted bits. In this section BER analysis of OOK, BPPM and BPSK is done. For binary modulation schemes the BER  $P_b$  is directly linked with PEP (posterior error probability)  $P_e(d)$  which depends only on Euclidean distance. The Euclidean distance between the constellation points  $s$  and  $\hat{s}$  is denoted by  $d$ , the mathematical expression of  $d=\|s-\hat{s}\|$  and the PEP  $P_e(d,I)=Q(\sqrt{\gamma d^2 I^2 / (4\sigma^2)})$  which depends on SNR that is denoted by  $\gamma$  and irradiance  $I$ . From this formula, the PEP over a turbulent media has been derived, as given in equation 8.

The PEP over the turbulent channel is:

$$P_e(d) = Q\left(\sqrt{\frac{\gamma d^2}{4\sigma^2}} I\right) f(I) dI \tag{8}$$

Where  $\gamma$  is SNR and  $d$  represents the Euclidean distance for modulation techniques and  $f(I)$  is the PDF for both the models. The performance is analysed for OOK (on-off keying) which is a basic form of ASK (amplitude shift keying) in which for binary 1 the carrier is present and for binary zero there is no carrier. The comparison of OOK is presented with BPPM which is binary pulse position modulation; for this technique, the two bits in a symbol are used to transmit the data and with BPSK (binary phase shift keying) in which two symbols are 0 and 1 at phase of 180 degree are considered.

**Diversity and combining Gain.** The diversity combining gains is a technique to extract the information from various transmitted signals over different paths. This method gives a single improved signal by combining the various signals. To characterize a turbulence fading channel the terms diversity gain  $G_d$  and combining gain  $G_c$  are used. The diversity and combining gain for SISO Gamma-Gamma turbulence channel is obtained in equation 9 and 10 given by Bayaki, Schober and Mallik (2009):

$$G_d = \min(\alpha/2, \beta/2) \tag{9}$$

$$G_c = \left(\frac{d}{2\sqrt{2}\alpha\beta\sigma}\right)^2 X \left(\frac{2\sqrt{\pi}(\max\{\alpha,\beta\})\Gamma(2G_d+1)}{\Gamma(|\alpha-\beta|)\Gamma(G_d+\frac{1}{2})}\right)^{1/G_d} \tag{10}$$

For k-distribution we consider the  $\beta=1$  in diversity and combining gains and the final equations for gains are

$$G_d = \min(\alpha/2, 1/2) \tag{11}$$

$$G_c = \left(\frac{d}{2\sqrt{2}\alpha\sigma}\right)^2 X \left(\frac{2\sqrt{\pi}(\max\{\alpha,1\})\Gamma(2G_d+1)}{\Gamma(|\alpha-1|)\Gamma(G_d+\frac{1}{2})}\right)^{1/G_d} \tag{12}$$

In this section the mathematical analysis for MIMO FSO system is done. The MIMO system is known as multiple input and multiple output and used in wireless communication. The multiple antennas at both ends are used to reduce the errors and to increase the transmission speed.

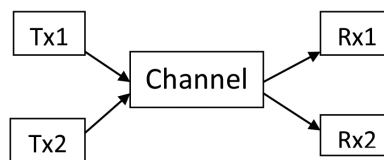


Figure 2. MIMO diversity (2x2)

### Performance Analysis of MIMO FSO Channel

**MIMO Gamma-Gamma channel.** The probability density function (PDF) of irradiance ( $I$ ) in Gamma-Gamma distribution in MIMO system is written as in equation 13 given by Luong and Pham (2014), where  $I$  is the function of  $M$  and  $N$  and denoted by  $I(MN)$ :

$$f(I(M, N)) = \frac{2(\alpha_1\beta_1/MN)^{\frac{(\alpha_1+\beta_1)}{2}} I^{(\alpha_1+\beta_1)/2-1} K_{\alpha_1-\beta_1}(2\sqrt{I\alpha_1\beta_1/MN})}{\Gamma(\alpha_1)\Gamma(\beta_1)}, I>0 \tag{13}$$

Where  $\alpha_1=MN\alpha$  and  $\beta_1=MN\beta$  are the new shaping parameters for MIMO Gamma-Gamma model. The numbers of transmitters are represented by  $M$  and where  $N$  is number of receive apertures. The case in which the  $M$  and  $N$  is 1, represents the SISO case as given in equation 3.

**MIMO K- distribution channel.** The probability density function of  $I(MN)$  for K-distribution in MIMO system is given in equation 14.

$$f(I(MN)) = \frac{2(\alpha_1/MN)^{\frac{(\alpha_1+1)}{2}} I^{(\alpha_1-1)/2} K_{\alpha_1-1}(2\sqrt{I\alpha_1/MN})}{\Gamma(\alpha_1)}, I>0 \tag{14}$$

### BER and Gain performance in MIMO

1) *BER Analysis:* For MIMO BER analysis the PDF  $f(I(MN))$  from equation 13 and 14 is considered for the formula given in equation 8 for both Gamma-gamma and K-distribution. The final BER for MIMO system is given in equation 15:

$$P_e(d) = Q \left( \sqrt{\frac{\gamma d^2}{4\sigma^2}} I^2 \right) f(I(MN)) dI \tag{15}$$

2) *Diversity and Combining Gains*: The diversity gain and combining gain for MIMO Gamma-Gamma and K- channel is given in equation 16 as explained by Tsiftsis et al. (2009).

$$G(M, N) = \frac{G_C^{MRC}}{G_C^{EGC}} = N \left( \frac{2\Gamma(2MG_d)}{\Gamma(MG_d)} \right)^{1/MG_d} \times \left( \frac{\Gamma(MNG_d+1)}{\Gamma(2MNG_d+1)} \right)^{1/MNG_d} \tag{16}$$

### RESULTS

In this section the analysis of BER and Gain of Gamma-Gamma and K-distribution channel is presented for SISO and MIMO systems.

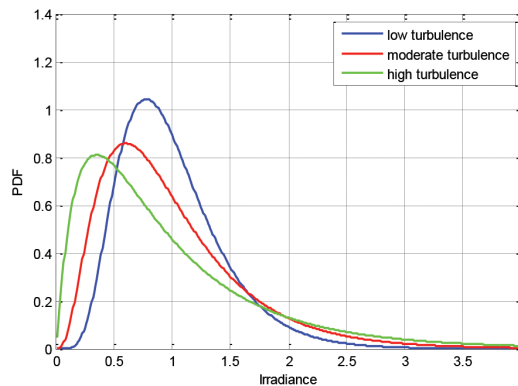


Figure 3. Probability density function vs. irradiance for Gamma-gamma model

Figure 3 represents the variation of probability density function with respect to irradiance for Gamma-Gamma turbulent channel under different turbulence regimes assuming the contribution of both large and small turbulence eddies and it has been shown that the PDF decreases with increase in turbulence. In Figure 4 the PDF vs. irradiance graph is presented for K-channel model for different turbulence scenarios. It has been shown that as the value of (alpha) which is no. of discrete scatters decreases the turbulence increases and the PDF decreases. The value for PDF only occurs at negative slope as in negative exponential distribution. This represents that K-distribution has very high turbulence condition.

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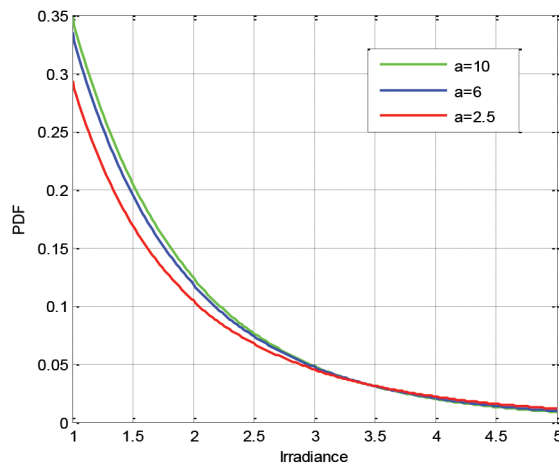


Figure 4. Probability density function vs. irradiance for K-distribution model

Table 1  
Parameters and PDF for different turbulence regimes

Turbulence	$\alpha$ (G-G)	$\beta$ (G-G)	$\alpha$ (k-dist.)	PDF (G-G)	PDF (k-dist.)
Weak	11	10	10	0.9	0.14
Moderate	7.1	4.5	6	0.65	0.12
Strong	4.4	4.2	2.5	0.45	0.1

In Table 1 the values for different parameters of both Gamma-Gamma and K-distribution are given and the PDF for Gamma-Gamma at irradiance =1 for all turbulence regimes is included in this table, where PDF at irradiance =2 is included for K-distribution model.

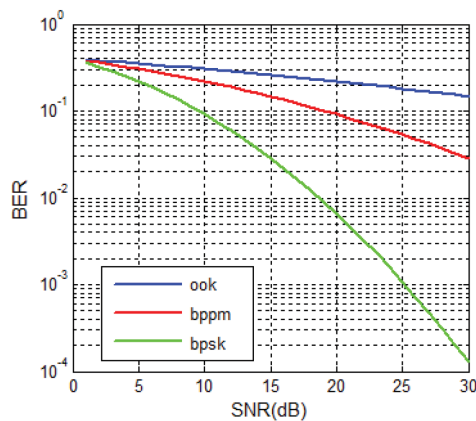


Figure 5. BER vs. SNR for SISO Gamma-gamma model under different modulation techniques

In Figure 5 the BER vs. SNR graph is presented for different modulation schemes and it can be concluded that the BPSK modulation gives appreciable results in comparison of other binary modulation techniques for SISO Gamma-Gamma model. As shown in graph in order to achieve the BER of order  $10^{-1}$  the corresponding SNR for BPSK is 9.8dB and for BPPM the SNR is 19.7dB which shows that BPSK performs better.

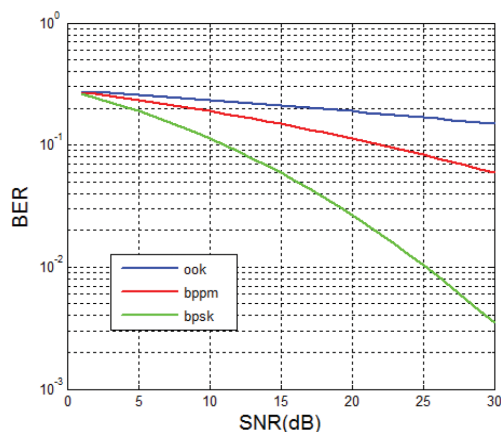


Figure 6. BER vs. SNR for SISO K-distribution model under different modulation techniques

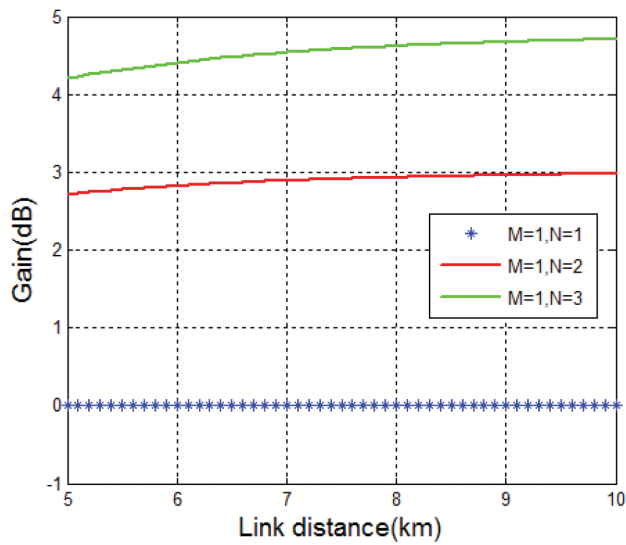
The Figure 6 represents the performance of K-distribution with respect to BER vs. SNR for different modulation techniques. The performance of BPSK is better from other techniques. To achieve  $BER = 10^{-1}$ , the SNR required for BPPM is 21dB where for BPSK the SNR is 10.8dB. In comparison to Gamma-Gamma channel model for same modulation technique the K-distribution requires large SNR which concludes that Gamma-Gamma channel model is better and all the parameters are presented in table 2 below.

Table 2

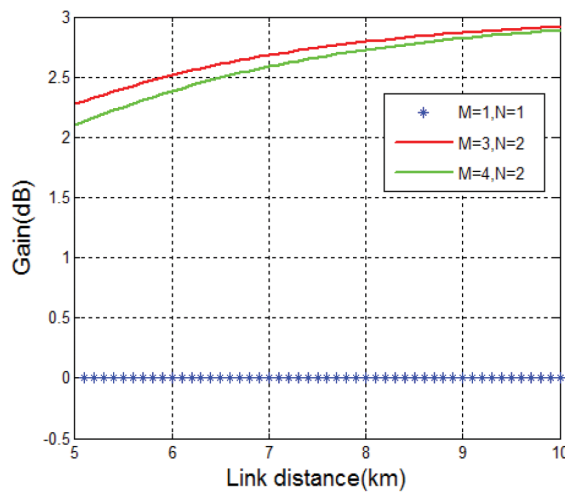
Comparison of different modulation schemes for SISO Gamma-Gamma and K-distribution channel

Modulation	SNR for BER	SNR for BER
	$10^{-1}$ (Gamma-Gamma)	$10^{-1}$ (K-distribution)
<b>BPSK</b>	9.8dB	10.8dB
<b>BPPM</b>	19.7dB	21dB
<b>OOK</b>	More than 30dB	More than 30dB





(a)



(b)

Figure 7. Plot of Gain with Link distance for Gamma-gamma model (a) Receiver diversity (b) Transmit diversity

In Figure 7 the performance of diversity systems in terms of gain is analysed and the conclusion drawn indicates the gain of system increases as the number of receiver aperture (receiver diversity) increases. In Figure 7(a) the system which has  $M=1, N=3$  diversity gives improved gain up to 4.2dB where low diversity order has Gain=2.8dB. In Figure 7(b) the transmitter diversity system is considered and concluded that by increasing the number of transmitters the performance of gain decreases, the exact results shown in Table 3.

Table 3  
*Performance comparison of Gain for different diversity systems in Gamma-Gamma channel model*

Diversity	Gain at L=6km	Gain at L=9km
M=1,N=2	2.9dB	3db
M=1,N=3	4.4dB	4.8dB
M=3,N=2	2.5dB	2.9dB
M=4,N=2	2.35dB	2.89dB

The plot in Figure 8 is presenting the comparison analysis of gain vs. link distance for gamma-gamma channel model and K-distribution. The performance of gamma-gamma in SIMO (M=1, N=3) diversity has better result than SISO. Where the K-distribution with same SIMO diversity has less gain than Gamma-Gamma distribution as given in Table 4.

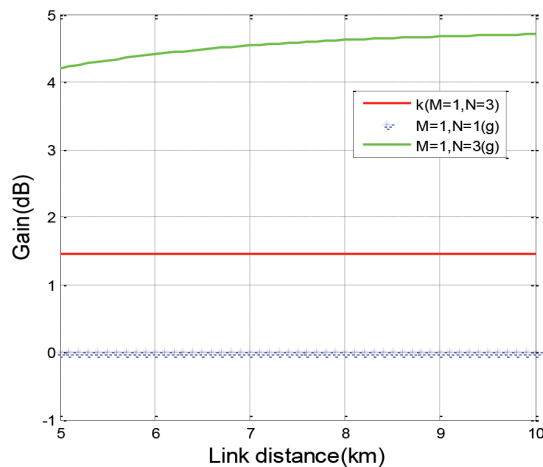


Figure 8. Comparison of Gain vs. Link distance for Gamma-gamma and K-distribution

Table 4  
*Gain comparison for Gamma-Gamma and K-distribution*

Channel Model	Diversity	Gain(dB) at L=6km	Gain(dB) at L=9km
Gamma- Gamma	M=1,N=3	4.4	4.8
K- distribution	M=1,N=3	1.5	1.5

Table 5 is based on Figure 9 in which the MIMO Gamma-Gamma channel model with OOK and BPPM modulation is presented. The BER calculated for M=1 and N=3 diversity system. To achieve the BER =  $10^{-3}$  the SNR for BPPM is required up to 37.5dB where for OOK it is more than 40dB.

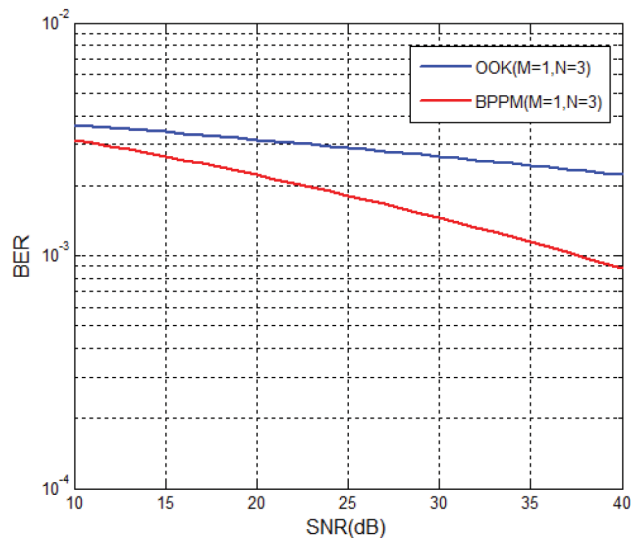


Figure 9. BER vs. SNR for MIMO Gamma-Gamma channel model with modulation

Table 5

Performance comparison for MIMO Gamma-Gamma model for OOK and BPPM

Channel Model	Diversity	Modulation	SNR at BER ( $10^{-3}$ )
Gamma- Gamma	M=1,N=3	OOK	>40dB
	M=1, N=3	BPPM	37.5dB

In Figure 10 the comparison for BPPM and BPSK is carried out for M=2 and N=2 diversity order. It is analysed that to achieve BER=  $10^{-5}$  the required SNR for BPSK is 25.1dB where for BPPM it is more than 40dB. Thus, it can be concluded that BPSK performs better than BPPM and BPPM performs better than OOK. Table 6 is based on Figure 10 allowing us to conclude that the BPSK is a better technique.

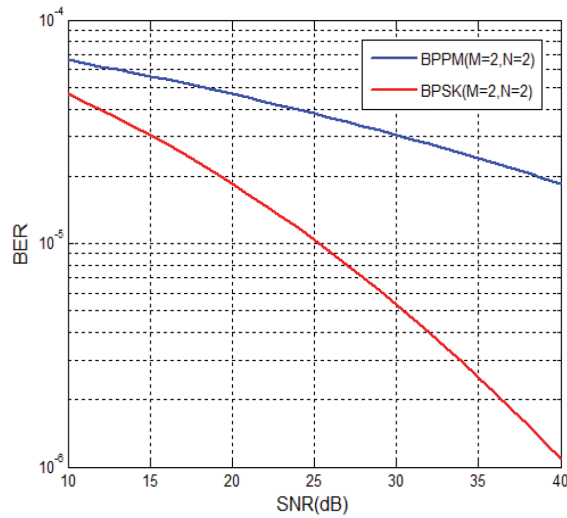


Figure 10. BER vs. SNR for MIMO Gamma-Gamma channel model with modulation

Table 6

Performance of MIMO Gamma-Gamma model for BPPM and BPSK

Channel Model	Diversity	Modulation	SNR at BER ( $10^{-5}$ )
<b>Gamma-</b>	M=2, N=2	BPPM	>40dB
<b>Gamma</b>	M=2, N=2	BPSK	25.1dB

Figure 11 represents the performance of MIMO K-distribution channel with OOK and BPPM modulation techniques. The performance analyses presented for OOK and BPPM with M=1 and N=3 diversity order the SNR required to achieve BER =  $10^{-2}$  is more than 40dB in case of OOK modulation where for BPPM SNR is 29.6dB.

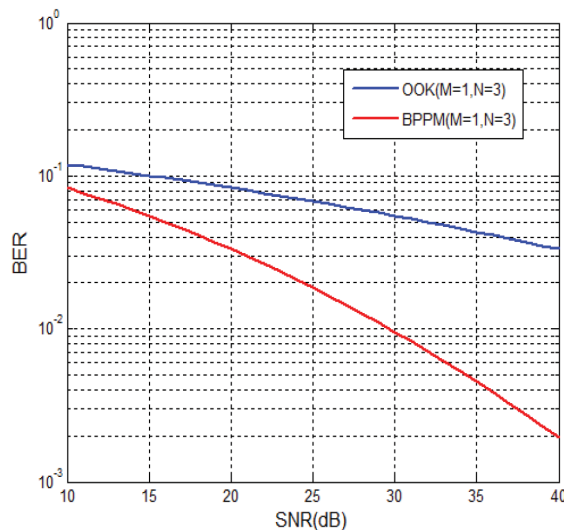


Figure 11. BER vs. SNR for MIMO K-distribution channel model with modulation

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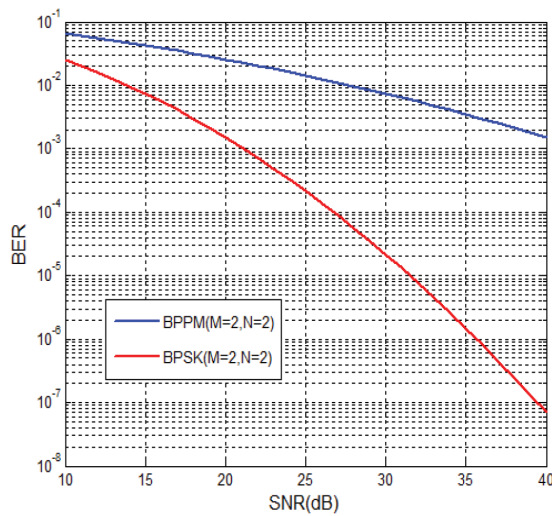


Figure 12. BER vs. SNR for MIMO K-distribution channel model with modulation

The Table 7 is based on Figure 11 in which the performance of K-distribution is analysed for MIMO system by considering OOK and BPPM diversity and concluded that BPPM performs better.

Table 7  
Performance of MIMO K-distribution model for OOK and BPPM

Channel Model	Diversity	Modulation	SNR at BER ( $10^{-2}$ )
<b>K-distribution</b>	M=1, N=3	OOK	>40dB
	M=1, N=3	BPPM	29.6dB

On basis of Figure 12 the Table 8 is presented in which two modulation techniques BPPM and BPSK are analysed over K-distribution channel model by considering M=2 and N=2 diversity order and it is analysed that to achieve BER =  $10^{-3}$  the SNR required for BPPM modulation is >40dB and with BPSK 21dB.

Table 8  
Performance of MIMO K-distribution model for BPPM and BPSK

Channel Model	Diversity	Modulation	SNR at BER ( $10^{-3}$ )
<b>K-distribution</b>	M=2, N=2	BPPM	>40dB
	M=2, N=2	BPSK	21dB

## DISSCUSION

In this paper, Gamma-Gamma and K-distribution channel models were analysed for SISO and MIMO FSO links with the objective of determining the impact of turbulence on optical wireless link. The analysis was done by determining the Gain and BER performance of Gamma-Gamma and K-distribution. The probability density function of irradiance is also plotted and it is concluded that as the turbulence increases the pdf decreases for both the channels as in Gamma-Gamma for large turbulence the PDF at intensity 1 is 0.45 and for small turbulence 0.9. Similarly, for K-distribution the PDF for large turbulence at intensity 2 is 0.1 and for small 0.14. It is concluded that the BER for MIMO system is less than other diversities in both channel models but Gama-Gamma model gives better results than K-model. Analysis of the gain with link distance for both the channel models showed that the gain for Gamma-Gamma channel is 4.4dB where for K- distribution model it is 1.5dB for same diversity order. By comparing the different diversity orders, it is shown that the Gamma-Gamma channel performs better than K-distribution in all types of turbulence scenarios.

## CONCLUSION

There are some digital modulation techniques OOK, BPPM, BPSK are also compared in this paper and it is concluded that the BPSK performance better than others. The Gamma-Gamma channel model is declared as better model than K-model.

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